NOISE-INDUCED PHENOMENA IN THE ENVIRONMENTAL SCIENCES

Randomness is ubiquitous in nature. Random drivers are generally considered a source of disorder in environmental systems. However, the interaction between noise and nonlinear dynamics may lead to the emergence of a number of ordered behaviors (in time and space) that would not exist in the absence of noise. This counterintuitive effect of randomness may play a crucial role in environmental processes. For example, seemingly “random” background events in the atmosphere can grow into larger instabilities that have great effects on weather patterns. This book presents the basics of the theory of stochastic calculus and its application to the study of noise-induced phenomena in environmental systems. It will be an invaluable reference text for ecologists, geoscientists, and environmental engineers interested in the study of stochastic environmental dynamics.

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Noise-induced phenomena are characterized by the ability of noise to induce order (either in space or in time) in dynamical systems. These phenomena are caused by the randomness of external drivers, and they would not exist in the absence of noise. The ability of noise to create order is counterintuitive. In fact, until recently, noise was generally associated with disordered random fluctuations around the steady states of the underlying deterministic dynamics. However, in the past few years the scientific community has become aware that noise can also have a more fundamental effect, in that it can determine new states and new dynamical patterns.

The speculative “beauty” of these dynamical behaviors, as well as the ubiquitous occurrence of random drivers in a number of natural and engineered systems, explains the great attention that has been recently paid to the study of noise-induced phenomena. A number of recent contributions have shown that the emergence of order and patterns in nature may result as an effect of the noise inherent in environmental variability. A typical example is climate fluctuations and their ability to induce dynamical behaviors that would not exist in the absence of random climate variability.

The main reason for writing this book is that there is a rich body of literature on noise-induced phenomena in the environmental sciences, and it has become difficult to keep track of the main theories, methods, and findings that have been presented in a number of research articles spread throughout the physics, mathematics, geoscience, and ecology journals. After working for a few years in this research field, we have become aware of the need for a book that (1) describes the main mechanisms of noise-induced order in space and in time; (2) presents rigorous mathematical tools addressing a relatively broad readership of environmental scientists, who are not necessarily familiar with the theory of stochastic processes; (3) focuses on applications to the environmental sciences; and (4) reviews a number of recent studies on noise-induced phenomena in environmental dynamics.

The goal of this book is to provide a synthesis of theories and methods for the study of noise-induced phenomena in the environment and to draw the attention of the
earth and environmental science communities toward this fascinating and challenging research area. Through a number of examples of noise-induced phenomena we stress how in the natural environment random fluctuations are the rule and interesting behaviors may emerge from the interactions between the deterministic and stochastic components of environmental dynamics.

This book is not intended to be a comprehensive treatise on noise-induced phenomena. This relatively vast and fast-moving research field is enriched every day with new studies appearing in the literature. It would not be possible to contain in this volume an exhaustive review of all the existing theories of noise-induced order and their application to the environmental sciences. This book tries to provide an organized synthesis of the main contributions to this subject, drawing from material that is currently spread through a number of journals and other publications.

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